Modal Circuit Decomposition of Lossy Multiconductor Transmission Lines

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***Abstract*—General multi-conductor transmission lines are investigated using modal analysis. This is performed by finding the solution to the telegrapher's equations for general impedance and admittance per unit length matrices Z and Y, respectively, and obtaining the transmission matrix in terms of Z and Y. Hence, the modal circuit is sought, resulting in a cascade of two n-port ideal transformers and n uncoupled transmission lines. A set of necessary and sufficient conditions are established and a construction method is given if the conditions are satisfied. It is shown that the modal circuit will always exist for general homogeneous constant parameters and for the nonhomogeneous case under the quasi-TEM assumption the existence depends on the geometry. The modal circuit is extended for frequency dependent parameters and a set of sufficient conditions are given.**

I. Introduction

The modal circuit for multi conductor lines has been in use for some time now; it was analyzed in Uchida [1] in 1967 for various homogeneous lossless cases. Since then it has been used in SPICE [2] to simulate coupled lines. Chang [3] used it with the method of characteristics for the transient analysis of coupled lines. However, there is no detailed study for the necessary and sufficient conditions for the existence of the modal circuit. Chang proved the existence of the modal circuit for lossless nonhomogeneous lines in 1970 [4]. Chang [3] also proved the existence of the model for homogeneous lossy coupled lines with the extra assumption that the resistance matrix R is diagonal. It turns out that this is not necessary for the existence of the modal circuit. This paper aims to provide the analysis of the modal circuit and to obtain the necessary and sufficient conditions for its existence. This paper is divided into five sections. In Section II, the telegraphers equations are set up for the case to be studied and the solution, which is the transmission matrix, is obtained. In Section III, the decoupling of the transmission matrix is investigated and a set of necessary and sufficient conditions is obtained. Section IV presents special cases where decoupling is investigated. Finally, Section V provides the conclusion.

II. Telegraphers Equations

The Telegrapher's equations for the transmission line shown in Fig. 1 are given as follows:

$$\frac{dV}{dx} = -ZI \tag{1}$$

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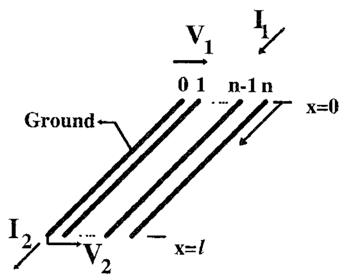

Fig. 1 General coupled transmission line.

$$\frac{dI}{dx} = -YV \tag{2}$$

$$Z = R + j\omega L,$$
$$Y = G + j\omega C \tag{3}$$

where, R, L, C, and G are parameters per unit length. Physically the coupled line structure is taken to have $n+1$ lines numbered 0 to n as in Fig. 1 with line number 0 taken as ground. The voltage $V_i(x)$ is defined as the difference of absolute potentials $\phi_i(x) - \phi_o(x)$, where the potential $\phi(x)$ along the line is taken relative to zero voltage at infinity or any other convenient point. This formulation allows the inclusion of lossy and lossless ground lines in all of the following analysis. For now, the only assumptions are that matrices R, L, C, and G are bounded real matrices (at any frequency) and independent of distance x. This is the only assumption needed to obtain the transmission matrix for the given structure. The solution is obtained by setting up the $2n$ first order differential equations using the vector P defined as,

$$P(x) = \begin{bmatrix} V(x) \\ I(x) \end{bmatrix}. \tag{4}$$

The orientation of voltages and currents are as shown in Fig. 1. Hence, (1) and (2) can be rewritten as

$$\frac{dP}{dx} = -MP \tag{5}$$

where

$$M = \begin{bmatrix} 0 & -Z \\ -Y & 0 \end{bmatrix}. \tag{6}$$

This system of (5), has a solution represented as an exponential power series that will converge for all x (see Appendix A).

The solution can be written as in (7), shown at the bottom of the page. From the definition of the problem we now have

$$P(0) = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}, P(l) = \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$
$$P(l) = \exp(Ml)P(0). \tag{8}$$

Therefore, the transmission matrix for the complete coupled line structure, call it A, is exactly the inverse of matrix $\exp(Ml)$. The inverse is achieved using the usual exponential identity which holds for matrices [5],

$$\begin{aligned} \exp(Mx)\exp(-Mx) &= \exp[M(x-x)] = 1_n \\ \Rightarrow \exp^{-1}(Mx) &= \exp(-Mx). \end{aligned} \tag{9}$$

Note: 1_n is $n \times n$ identity matrix. This result is obtained with no diagonalization so that no unnecessary conditions are implied in the result. The result is re-written by changing the indexing variable

$$A = \begin{bmatrix} \sum_{k=0}^{\infty} \frac{(ZY)^k l^{2k}}{(2k)!} & \sum_{k=0}^{\infty} \frac{(ZY)^k l^{2k+1}}{(2k+1)!} Z \\ Y \sum_{k=0}^{\infty} \frac{(ZY)^k l^{2k+1}}{(2k+1)!} & \sum_{k=0}^{\infty} \frac{(YZ)^k l^{2k}}{(2k)!} \end{bmatrix}. \tag{10}$$

This expression can be compared to previous results [6] if we assume that there is a matrix γ such that $\gamma^2 = ZY$. Usually, γ is the result of eigen analysis of the ZY matrix. Using the series representation of cosh and sinh functions we may represent (10) as

$$A = \begin{bmatrix} \cosh(\gamma l) & \sinh(\gamma l) Z_w \\ Y_w \sinh(\gamma l) & \cosh^t(\gamma l) \end{bmatrix} \tag{11}$$

where

$$\begin{aligned} Z_w &= \gamma^{-1} Z, \\ Y_w &= Y \gamma^{-1}. \end{aligned} \tag{12}$$

This is the usual expression for the transmission matrix of a coupled transmission line. However, here it is proved that it applies for the general case of a nondiagonalizable ZY matrix. Another approach to reach this result was followed by Faria [6] using the Jordan Canonical Form of matrix ZY.

III. Decoupling into the Modal Circuit

Tracing the origins of the modal method there are principally two approaches of achieving the decoupling of the multiconductor coupled lines. The usual mathematical approach is based on matrix theory and the eigen analysis of matrices. The other approach is based on physical reasoning as presented in [7] for lossless homogeneous coupled lines. The latter approach decouples the power transferred on the multi conductor line into n separate excitations. We will show that both methods lead to the same set of conditions when applied to the general case as in (1) and (2). The final part of this section will address the question of frequency dependent parameters.

A. Eigen Analysis

The modal method of decoupling the multiconductor transmission line is based on $n \times n$ ideal transformers appended on both sides of the transmission line, to model coupling, and n uncoupled single lines as shown in Fig. 2. The transmission matrix for the ideal transformer can be found from the conservation of power between input and output and the independence of voltages and currents. Let,

$$A = \begin{bmatrix} T_v & 0 \\ 0 & T_i \end{bmatrix},$$

$$\begin{aligned} P_{in} &= P_{out} \\ \Rightarrow V_1^t I_1^* &= V_2^t I_2^* \\ \Rightarrow V_2^t T_v^t T_i I_2^* &= V_2^t I_2^*, \quad \forall V_2, I_2 \\ \Rightarrow T_v^t T_i &= 1_n \\ \Rightarrow T_v &= T, \\ T_i &= T^{t^{-1}} \end{aligned} \tag{13}$$

where (*) is the complex conjugate. For the circuit as in Fig. 2 to model the coupled transmission lines we have,

$$\begin{aligned} A_{coupled} &= A_t A_{uncoupled} A_t^{-1} \\ \Rightarrow A_{uncoupled} &= A_t^{-1} A_{coupled} A_t. \end{aligned} \tag{14}$$

Hence, $A_{uncoupled}$ is

$$\begin{bmatrix} T^{-1} \sum_{k=0}^{\infty} \frac{(ZY)^k l^{2k}}{(2k)!} T & T^{-1} \sum_{k=0}^{\infty} \frac{(ZY)^k l^{2k+1}}{(2k+1)!} Z T^{t^{-1}} \\ T^t Y \sum_{k=0}^{\infty} \frac{(ZY)^k l^{2k+1}}{(2k+1)!} T & T^t \sum_{k=0}^{\infty} \frac{(YZ)^k l^{2k}}{(2k)!} T^{t^{-1}} \end{bmatrix}. \tag{15}$$

For the model to exist the matrix in (15) must consist of four diagonal matrices. This leads to the following set of necessary and sufficient conditions. For justification for the necessity and sufficiency of these conditions see Appendix B.

$$T^{-1} ZYT = D_1 \tag{16}$$
$$T^{-1} Z T^{t^{-1}} = D_2 \tag{17}$$
$$T^t Y T = D_3 \tag{18}$$
$$T^t Y Z T^{t^{-1}} = D_4. \tag{19}$$

$$P(x) = \exp(Mx)P(0) = \begin{bmatrix} \sum_{n=0,\ n \text{ even}}^{\infty} \frac{(ZY)^{n/2} x^n}{n!} & -\sum_{n=0,\ n \text{ odd}}^{\infty} \frac{(ZY)^{(n-1)/2} x^n}{n!} Z \\ -Y \sum_{n=0,\ n \text{ odd}}^{\infty} \frac{(ZY)^{(n-1)/2} x^n}{n!} & \sum_{n=0,\ n \text{ even}}^{\infty} \frac{(YZ)^{n/2} x^n}{n!} \end{bmatrix} \cdot P(0) \tag{7}$$

Where, D_1, D_2, D_3, and D_4 are diagonal matrices. These conditions are sufficient and necessary for the existence of the modal circuit. However, they are not independent and can be reduced to just (17) and (18) since if they are satisfied then (16) and (19) are automatically satisfied for Z and Y symmetric. Therefore, (17) and (18) are necessary and sufficient for the existence of the modal equivalent circuit if T is also real and independent of frequency. This can be incorporated in the equations if we expand Z and Y in terms of R, L, C, and G matrices, using the fact that (17) and (18) should be satisfied for zero and close to infinite frequencies, leading to the set of conditions.

$$T^{-1}RT^{t^{-1}} = D_7 \tag{20a}$$
$$T^{-1}LT^{t^{-1}} = D_8 \tag{20b}$$
$$T^tGT = D_9 \tag{20c}$$
$$T^tCT = D_{10}. \tag{20d}$$

Thus the existence of real nonsingular matrix T to satisfy (20) is a necessary and sufficient condition for the existence of the modal circuit. This is the main result of this paper, that is, the physical existence of the modal circuit is transformed into a set of mathematical conditions that can be simplified using matrix theory. To simplify the testing procedure of conditions (20) the properties of matrices R, L, C, and G have first to be established. It can be shown that matrices L and C are nonsingular [4]. For matrices R and G that model different mechanisms for loss in the multi-conductor line, they are necessarily strictly positive definite i.e., nonsingular. Thus conditions (20) are transformed into simpler equivalent conditions which can be easily checked for any arbitrary case. The proof is provided in Appendix C.

$$CRC_o = C_oRC \tag{21a}$$
$$GRC_o = C_oRG \tag{21b}$$
$$CRG = GRC. \tag{21c}$$

In obtaining conditions (21) the symmetry of all R, L, C, and G matrices was used and the fact that L and R are nonsingular. Note that (21) are symmetric in R, L, C, and G matrices since all of them are nonsingular. The above form (21) is chosen as it is the most convenient form. Since all matrices are symmetric, the conditions are equivalent to all RHS or LHS products being symmetric. The conditions (21) are obtained using the fact that the conductor loss is present and the matrix R is nonsingular. However, if the case arises that only dielectric loss is present without conductor loss, the sufficient and necessary condition becomes

$$CLG = GLC. \tag{22}$$

Of course, this condition is implicit in (21) when R is present.

B. Power Decoupling

The simplest approach to present this method is to outline the analysis for the lossless homogeneous case given in [7] and then generalize the result to the general case in (1) and (2).

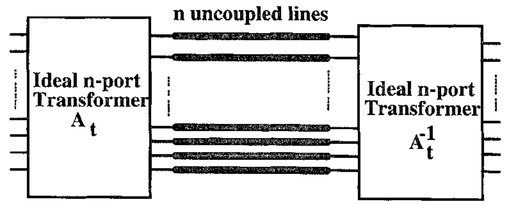

Fig. 2. Modal circuit of n decoupled lines

As in [7] for the lossless homogeneous case the telegraphers equations, for the structure in Fig. 1 reduce to

$$\frac{d^2V}{dx^2} + \omega^2 LCV = 0 \tag{23}$$
$$\frac{d^2I}{dx^2} + \omega^2 CLI = 0 \tag{24}$$

where

$$LC = \mu\epsilon 1_n. \tag{25}$$

From (23)–(25) the solution is found to be of the usual form of $\exp(\pm jbx)$ where, $b^2 = \mu\epsilon\omega^2$. Applying (1) and (2) to the positive going wave [i.e., $\exp(-jbx)$ only] we can relate voltages and currents

$$I = \frac{1}{\sqrt{\mu\epsilon}} CV \tag{26}$$
$$V = \frac{1}{\sqrt{\mu\epsilon}} LI. \tag{27}$$

Following the basic idea in [7], assume that there are n excitations (modes of propagation) of voltages $V_1, \cdots, V_n$, and currents $I_1, \cdots, I_n$, and terminal voltages and currents are a superposition of these modes, i.e.,

$$V = \sum_{j=0}^{n} v_{mj} V_j$$
$$I = \sum_{j=0}^{n} i_{mj} I_j. \tag{28}$$

Where, V_j and I_j are the modal excitations for mode j and v_{mj} and i_{mj} are scalar factors (possibly complex) of superposition. The modes are taken as orthogonal if,

$$V_i^t I_j^* = 0, \quad \forall i \neq j. \tag{29}$$

From (28) and (29) we can deduce the following:

$$P = V^t I^*$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} v_{mi} i_{mj}^* V_i^t I_j^* \tag{30}$$
$$= \sum_{i=1}^{n} v_{mi} i_{mi}^* V_i^t I_i^*$$
$$= \sum_{i=1}^{n} P_i \tag{31}$$

taking

$$P_i = v_{mi} i_{mi}^* \tag{32}$$
$$\Rightarrow V_i^t I_j^* = \delta_{ij}. \tag{33}$$

We can use (26), (27) to find the equations defining V_i, I_i. using (33),

$$V_i^t C V_j^* = \sqrt{\mu\epsilon}\delta_{ij} \quad (34)$$

$$I_i^t L^t I_j^* = \sqrt{\mu\epsilon}\delta_{ij}. \quad (35)$$

Writing the result in matrix form, and using the fact that L and C are real symmetric matrices, in which case T and F will also be real.

$$T^t C T = D \quad (36a)$$

$$F^t L F = D \quad (36b)$$

$$T^t F = 1_n \quad (36c)$$

$$D = \sqrt{\mu\epsilon} 1_n \quad (36d)$$

where T is the matrix who's columns are I_i. Equation (36c) is actually (33) rewritten in matrix form. These equations actually represent a change of basis, or physically a superposition of independent modes, in which the capacitance and inductance matrices are represented by diagonal matrices i.e., no coupling. Now the above procedure is applied to the general cases as in (1) and (2).

$$\frac{d^2V}{dx^2} - ZYV = 0 \quad (37)$$

$$\frac{d^2I}{dx^2} - YZI = 0 \quad (38)$$

where Z and Y are defined as in (3). Assuming the existence of modes, as before, mode i propagates with a propagation constant of γ_i. In general, γ_i will depend on frequency. Then for the positive going wave for that mode we have, from (1) and (2)

$$V_i = \gamma_i^{-1} Z I_i \quad (39)$$

$$I_i = \gamma_i^{-1} Y V_i. \quad (40)$$

Applying the same power decoupling procedure as before we get

$$V_i^t Y^* V_j^* = \gamma_i^* \delta_{ij} \quad (41)$$

$$I_i^t Z^t I_j^* = \gamma_i \delta_{ij}. \quad (42)$$

We can write the result as in (36) by using the fact that the modes (excitations) should be independent of frequency and that R, L, C, and G are assumed to be constants with respect to frequency. Writing the result in matrix form, for R, L, C, and G real symmetric matrices

$$T^t C T^* = D_c \quad (43a)$$

$$T^t G T^* = D_g \quad (43b)$$

$$F^t R F = D_r \quad (43c)$$

$$F^t L F = D_l \quad (43d)$$

$$T^t F^* = 1_n. \quad (43e)$$

This is a direct generalization of (36). Hence, the problem of existence of the modal circuit reduces to the existence of matrices T and F such that (43) are satisfied. If there is a matrix T to satisfy (43) it will also be real, as shown in Appendix C. Hence, (43) are exactly the same as the conditions (20).

C. Frequency Dependent Parameters

For frequency dependent parameters the matrices L, C, R, and G depend on frequency in a nontrivial manner. The trivial case occurs when the frequency dependence can be factored out of the matrix resulting in a constant matrix with frequency dependent scalar. This case is part of the constant parameters as above. The model for the general case, still as in Fig. 2, is an ideal transformer and n uncoupled lines with frequency dependent parameters. Physically this case corresponds to the quasi-TEM approximation with ϵ_{eff}, inductance including internal inductance and loss that are frequency dependent. The above procedure can only provide sufficient conditions since the transition from conditions (17) and (18)–(20) will preserve the sufficiency but not in general the necessity of the conditions. Thus the set of conditions (20) provide sufficient conditions if they are satisfied for all frequencies with T a constant real matrix. A sufficient condition for the above statement is that every matrix commutes with itself and all other matrices for all frequencies. The proof is discussed in [8] and [9]. i.e.,

$$\begin{aligned} [R(\omega_1), R(\omega_2)], [C(\omega_1), C(\omega_2)], [G(\omega_1), G(\omega_2)] &= 0 \\ [L(\omega_1), R(\omega_2)], [L(\omega_1), C(\omega_2)], [L(\omega_1), G(\omega_2)] &= 0 \\ [C(\omega_1), R(\omega_2)], [C(\omega_1), C(\omega_2)], [R(\omega_1), G(\omega_2)] &= 0 \\ [L(\omega_1), L(\omega_2)] &= 0 \quad (44) \end{aligned}$$

$$\forall\, \omega_1, \omega_2 \text{ and } [A, B] = AB - BA.$$

These conditions are quite hard to check numerically and practically can only be applied in symbolic form. Nevertheless, there are cases where they can be applied.

IV. Special Cases

The conditions (21) and (44) allow a general, unified, investigation of the coupled lines. However, there are a number of special cases that need to be considered separately in order to note possible simplifications to conditions (21) and/or to compare with previous results.

A. Lossless Homogeneous

This case was the first to be studied in the context of coupled line structures since this is the only case where true TEM wave propagation exists. The other cases are approximations to this case using perturbation theory. In this case,

$$\begin{aligned} R &= 0, \\ G &= 0, \\ L &= \mu\epsilon C^{-1} \quad (45) \end{aligned}$$

then all that is required is to diagonalize the capacitance matrix C, which is symmetric, and we are guaranteed [17] to find a matrix T to satisfy (20a) which, because of (45) will then also satisfy (20b). Each capacitance per unit length value, C_i, for the n uncoupled transmission lines will be one of the eigenvalues of C, and hence real and positive. The inductance, $L_i = \epsilon\mu C_i^{-1}$, since the eigenvalues of the inverse of a matrix are the reciprocal of the eigenvalues of that matrix. In this case we end up with n lossless homogeneous uncoupled lines.

B. Lossless Nonhomogeneous

This case corresponds to the quasi-TEM approximation where the transmission line characteristics are close to those of a homogeneous line with dielectric permitivity ϵ_{eff} and with TEM wave propagation. This case is extensively covered in the literature since most of the practical structures for digital computers and microwave circuits, mostly microstrip geometry, are of this class. The general solution has been proven by Chang [4] who provides an explicit construction method. Thus the treatment of this case is mainly given as part of the general treatment of the coupled lines. In addition the method to find the matrix T for the model is achieved by a different numerical method using the Cholesky Factorization which is standard numerical procedure for such problems. In this case

$$\begin{aligned} R &= 0, \\ G &= 0, \\ L &= \mu\epsilon C_o^{-1} \end{aligned} \tag{46}$$

where C_o is the air capacitance,and we need to simultaneously congruence diagonalize C_o and C by a matrix T. This is done by taking the inverse of (20b) to get, using (46)

$$T^t C_o T = D_{11} \tag{47}$$

$$T^t C T = D_{12}. \tag{48}$$

It is shown that such a matrix exists (see Appendix C). Hence, each inductance value L_i for the n uncoupled transmission lines will be an eigenvalue of L. The C_i will be one of the eigenvalues of the matrix C. In this case we end up with n lossless nonhomogeneous uncoupled lines.

C. Lossy Homogeneous

This case corresponds to the quasi-TEM approximation to the TEM wave propagation using perturbation theory. However, if the loss is due to dielectric loss then the wave is still TEM with complex ϵ, and it will be subclass of case (A) rather than this case. This will be reflected in the analysis where a different problem will result only if matrix R is nonzero. For a discussion of electromagnetic wave propagation in transmission lines with conductor loss refer to Collins [10]. The modal circuit for this case is proven by Chang [3] under the assumption that R is diagonal, a condition not assumed in this paper. The quasi-TEM approximation for coupled transmission lines with lossy conductors is achieved by assuming that the transverse currents in the conductors are negligible compared with longitudinal currents. Hence, losses in the conductors due to the transverse currents are neglected and only losses due to longitudinal currents are considered. Therefore the resistance matrix R will be diagonal. However, in the case of lossy ground, the ground resistance will be added to each entry of the matrix R and it will not be diagonal. Chang's method could be extended by using $n+1$ conductors and ground at infinity, this, however, is an unnecessary increase of the order of the problem and can, in any case, be recovered from the present general analysis. In this case,

$$\begin{aligned} G &= \frac{\sigma}{\epsilon} C, \\ L &= \mu\epsilon C^{-1}. \end{aligned} \tag{49}$$

And we only need to simultaneously diagonalize R and L, with say T, where T will also satisfy (20c) and (20d) using (49). Then we only need to satisfy.

$$T^{-1} R T^{t^{-1}} = D_{13} \tag{50}$$

$$T^{-1} L T^{t^{-1}} = D_{14} \tag{51}$$

as in case (B) such a matrix T will exist, (Appendix C). Hence, in this case T can be used for the ideal transformer. For the parameters of the n uncoupled lines we can use the eigenvalues of the corresponding matrices. Note that this case is similar to (A) $L_i = \mu\epsilon C_i^{-1}$, where we have n equivalent lossy homogeneous lines. Also if only dielectric loss is present we only need to diagonalize L, or C, in order to achieve the modal circuit which is as in case (A).

D. Lossy Nonhomogeneous

This corresponds to the quasi-TEM approximation with approximations as in cases (B) and (C). Analysis in the literature has been for simple cases only, i.e., two conductors and to the authors knowledge there has never been a detailed study of the existence of the modal circuit. Unfortunately, in general there is no matrix T that satisfies (20). There exist pathological examples (see Appendix E) where even the matrix ZY is nondiagonalizable. Such a T can be found if and only if L, C, R, and G satisfy the relations (21). A construction method for T is provided for this case in Appendix C. If R, L, C, and G commute then (21a)–(21c) are satisfied and the resultant T will be orthogonal (see Appendix D). Hence, existence of the matrix T depends on the geometrical structure of the transmission line, and each structure should be treated separately. Numerically the test can be applied very easily utilizing a suitable error limit based on the precision of the evaluated parameters and the arithmetic used in calculations.

E. Perturbation Method

The perturbation method given by Harrington [11] is a further approximation within the quasi-TEM assumption, and leads in general to a modal equivalent circuit. This fact is due to the assumption that the losses are so small that they do not affect the imaginary part of the propagation constant, and the modal voltages and currents will be the same as in the lossless case. This assumption cannot be justified from general symmetric perturbation of the matrices in (52) and (53). If such perturbation is applied, both the eigenvalues (propagation constants) and the eigenvectors (the modal voltages and currents) will have linear dependence upon the perturbation. This is given in the theory of perturbation of linear operators as proven by Rellich [12]. Therefore the fact that the perturbation will have a lesser effect on the eigenvectors than the eigenvalues is taken as a postulate, rather than a result, based on practical and possible analytical justification arising from electromagnetic analysis of the general case. Under this postulate the perturbation of the coupled lines is equivalent to the perturbation of each of the uncoupled lines in the modal circuit separately. The method used in [11] is based on (41) and (42) rewritten here in matrix form for the general lossless case

$$C = M_v^{t^{-1}} V_p^{-1} M_v^{-1} \tag{52}$$

$$L = M_i^{t^{-1}} V_p^{-1} M_i^{-1} \tag{53}$$

$$M_v^t M_i = 1_n \tag{54}$$

where M_v and M_i are matrices whose columns are the modal voltages and currents, respectively. V_p is the diagonal matrix of propagation constants. Harrington obtained matrices L and C from solving the electrostatic problem to find C and C_o, C_o being the air capacitance of the same geometry. Then the matrices M_v and M_i are established through eigen analysis of the matrices L and C. This is always possible as shown in cases (A) and (B). Amari [13] suggested a way of finding the matrices L and C by first finding mode currents, M_i, and propagation constants, V_p, directly from the solution of the electromagnetic problem and then applying (52) and (53) to obtain L and C. However, equations in [13] contain transmitted power as a variable in each of the equations in (52) and (53). This however can always be cancelled out by using the superposition given in power decoupling analysis in Section II, or mathematically using the fact that the transmitted power is positive nonzero, and scalar multiplication of an eigenvector will also result in an eigenvector. Thus (52) and (53) present a more efficient method for finding matrices L and C for the lossless case. In either way (52) and (53) can always be obtained with V_p, M_v, and M_i real matrices. The attenuation constants for each mode are evaluated as in [11] using the modal currents and voltages from the following formulas:

$$\alpha_c = \frac{P_c}{2} \tag{55}$$

$$\alpha_d = \frac{P_d}{2} \tag{56}$$

where

$$P_t = V^{i^t} I^i = 1. \tag{57}$$

As above V^i, I^i are voltages and currents of mode i. P_c and P_d are the power loss per unit length due to conductors and dielectric loss, respectively and P_t is the transmitted power normalized as above. Following the analysis in [11] yields the following equations, (shown in Appendix F):

$$C = M_i V_p^{-1} M_i^t \tag{58a}$$

$$L = M_i^{t^{-1}} V_p^{-1} M_i^{-1} \tag{58b}$$

$$R = M_i^{t^{-1}} P_c M_i^{-1} \tag{58c}$$

$$G = M_i \; P_d \; M_i^t. \tag{58d}$$

Note that (58) are the same as conditions (20) since V_p, P_c, and P_d are diagonal matrices. Hence taking $T = M_v$ yields the modal circuit. In applying the above method, it is essential to assume that the real part of the propagation constant is much smaller than the imaginary part for every propagation mode. If the modal circuit is obtained using the parameters in (58) the propagation characteristics of the uncoupled lines will be different from those obtained by Harrington's method. This results from the assumption that loss will not affect the imaginary part of the propagation constants, an assumption which is assumed by Harrington and not in the current paper for the evaluation of the modal parameters. Thus using this method the real and imaginary parts of the propagation constants are

$$\beta_h = \frac{\omega}{v_p} \tag{59}$$

$$\alpha_h = \frac{P_c + P_d}{2} \tag{60}$$

while for the modal circuit they are

$$\beta_m = \sqrt{\frac{\|Z_m Y_m\| - \text{Real}\,(Z_m Y_m)}{2}} \tag{61}$$

$$\alpha_m = \frac{\text{Im}\,(Z_m Y_m)}{2\beta_m} \tag{62}$$

where

$$Z_m = R_m + j\omega L_m,$$
$$Y_m = G_m + j\omega C_m$$

are the immitances of mode m or equivalently the immitances of the mth uncoupled line. Hence using the dimensionless variables r and g as perturbation parameters defined as

$$r = \frac{R_m}{\omega L_m} \tag{63}$$

$$g = \frac{G_m}{\omega C_m} \tag{64}$$

we get the ratio of the two parameters as a power series in r and g [without $O(g^4)$ and $O(r^4)$ terms]

$$\frac{\beta_m}{\beta_h} = 1 + \frac{(g-r)^2}{8} + gr\,\frac{g^2 + \frac{gr}{2} + r^2}{32} - \frac{(gr)^3}{256}$$
$$\frac{\alpha_m}{\alpha_h} = 1 + \frac{(g-r)^2}{8} + gr\,\frac{3g^2 - \frac{5gr}{2} + 3r^2}{32} - \frac{13(gr)^3}{256} \tag{65}$$

which shows the correspondence of the two values for relatively low loss. The above procedure cannot unfortunately be extended to coupled lines given by the full wave analysis method as presented in [14] since cross mode powers are not, in general, negligible. Hence, for such cases conditions (21) or (44) apply.

F. Rotational Symmetry with Frequency Dependent Parameters and Each Conductor Coupled to All Other Conductors

The rotational symmetry is one of the rare cases where the problem can be solved, i.e., finding T, for the most general case of frequency dependent parameters without even

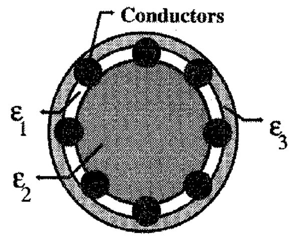

Fig. 3. Example of coupled lines with rotational symmetry.

determining the matrices R, L, C, and G. As shown in Fig. 3 the conductors can be of any shape, the only restriction is that rotational symmetry has to be observed. In this case the most general structure for matrices R, L, C, and G is a Toeplitz Symmetrical matrix, where the parameters of the matrix, i.e., $a_o \cdots a_{n-1}$, would be functions of frequency. The resultant matrix is of the form.

$$\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \cdots & a_{n-1} \\ a_1 & a_0 & a_1 & a_2 & \cdots & a_{n-2} \\ a_2 & a_1 & a_0 & a_1 & \cdots & a_{n-3} \\ a_3 & a_2 & a_1 & a_0 & \cdots & a_{n-4} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ a_{n-1} & a_{n-2} & a_{n-3} & a_{n-4} & \cdots & a_0 \end{bmatrix} \tag{66}$$

$$\text{where} \quad a_i = a_{n-i} \quad \forall\, i = 1, \cdots, n-1. \tag{67}$$

The condition (67) is due to the rotational symmetry. The advantage of such a structure is that all matrices of the form (66) commute with each other and there is an orthogonal matrix T (See Appendix G) that will diagonalize such matrix for any values of $a_o \cdots a_{n-1}$. The procedure of constructing T is given in [5].

G. Simple Microstrip Structure Without Edge Effects but with Frequency Dependence

This case represents the microstrip structure as in Fig. 4, with each conductor linked to its nearest neighbor, where the coupling parameters are independent of the conductor index but are frequency dependent. This case is suitable for printed circuit boards and coupled microstrip lines with a large number of conductors and weak coupling but dispersion is present. This case was analyzed for the case of lossless nonhomogeneous lines by Romeo [15] but the given procedure is valid for more general cases. The general R, C_o, C, and G matrices are of the form

$$\begin{bmatrix} A_0 & A_m & 0 & 0 & \cdots & 0 \\ A_m & A_0 & A_m & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 & 0 \\ \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & A_m & A_0 & A_m \\ 0 & \cdots & 0 & 0 & A_m & A_0 \end{bmatrix}$$

$$= A_0 1_n + A_m \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 & 0 \\ \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 & 1 \\ 0 & \cdots & 0 & 0 & 1 & 0 \end{bmatrix} \tag{68}$$

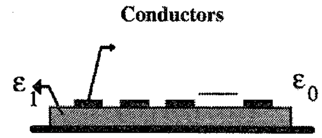

Fig. 4. Example of microstrip structure.

where A_o and A_m can be frequency dependent. All that is required in such a case is to diagonalize the last matrix with ones above and below the diagonal. The matrix T to diagonalize the above matrix is given in [15]. Hence, for such structure with possible frequency dependent parameters the modal circuit will exist and the uncoupled line will have frequency dependent parameters.

V. Conclusion

The necessary and sufficient conditions for the existence of the modal circuit for the case of constant parameters and sufficient conditions for the case of frequency dependent parameters are given. The model is shown to exist for a number of important practical configurations without additional conditions to the necessary conditions. A construction method is also provided if the conditions are satisfied to produce the parameters of the model which are the decoupling matrix and the parameters for the n uncoupled lines. The analysis presented here will facilitate derivation of other equivalent circuits for the coupled transmission lines with or without ideal transformers analogous to the lossless case. The model presented will also allow considerable improvements in numerical simulation and transient analysis. For using the method of waveform relaxation [3] it provides a set of necessary and sufficient conditions for its applicability and using convolutions as in [16] the number of convolutions per time step is reduced from $O(n^2)$ to $O(n)$. Further work is continuing in extending the model to wider applications and to find new models.

Appendix A

The series for $\exp(Mx)$ converges for all x and M complex bounded matrix of order $n \times n$.

The proof is given in [5]. The matrix exponential is evaluated from the fact that, for M as in (6)

$$M^p = \begin{cases} \begin{bmatrix} (ZY)^{p/2} & 0 \\ 0 & (YZ)^{p/2} \end{bmatrix} & p \text{ even} \\ \begin{bmatrix} 0 & -(ZY)^{(p-1)/2} Z \\ -Y(ZY)^{(p-1)/2} & 0 \end{bmatrix} & p \text{ odd} \end{cases}. \tag{69}$$

Appendix B

To prove conditions (16)–(19) the only non trivial step is that,

$$T \sum_{i=0}^{\infty} \frac{(ZY)^i l^i}{2i!} T^{-1} \tag{70}$$

is diagonal $\forall\, l$ if and only if matrix

$$TZYT^{t^{-1}} \tag{71}$$

is diagonal.

Proof: That (71) is sufficient for (70) is straightforward. Conversely since we have it for all l then choose $0 < l \ll 1$. Hence the term with $TZYT^{-1}$ will dominate and for the result to be diagonal this matrix has to be diagonal.

Appendix C

There exists matrix T such that for A, B, C, and E real, symmetric, matrices, A is strictly, positive definite, (nonsingular) matrix

$$T^t AT = D_a \quad (72a)$$
$$T^t BT = D_b \quad (72b)$$
$$T^t CT = D_c \quad (72c)$$
$$T^t ET = D_e \quad (72d)$$

if and only if A, B, C, and E satisfy

$$BA^{-1}C = CA^{-1}B \quad (73a)$$
$$BA^{-1}E = EA^{-1}B \quad (73b)$$
$$EA^{-1}C = CA^{-1}E. \quad (73c)$$

Proof: Here we will show that the above problem is equivalent to a simpler form and then use the result of Appendix D to show the equivalence of the conditions (73). The existence of T that satisfies (72) is equivalent to the existence of T_1 such that

$$T_1^t B' T_1 = D_{b'} \quad (74a)$$
$$T_1^t C' T_1 = D_{c'} \quad (74b)$$
$$T_1^t E' T_1 = D_{e'} \quad (74c)$$
$$T_1^t . T_1 = 1_n. \quad (74d)$$

Since A is real symmetric positive definite, there exists a Cholesky Factorization of A as in [17], call it U, which is upper triangular such that

$$A = U^t U. \quad (75)$$

U is nonsingular, since taking the determinant of both sides of (75) gives $\det(A) = \det^2(U)$. If U is singular then $\det(A) = 0$ which contradicts the assumption. And hence define

$$B' = U^{t^{-1}} BU^{-1} \quad (76)$$
$$C' = U^{t^{-1}} CU^{-1} \quad (77)$$
$$E' = U^{t^{-1}} EU^{-1}. \quad (78)$$

Now, if we have (72), Let

$$T_1 = UTD_a^{-1/2}. \quad (79)$$

Then substitute in (72) for T we get (73). Conversely, if we have (74) substituting for T_1 we get (72). Hence, as in Appendix D, T_1 that satisfies (74) exists if and only if B', C', and E' commute hence

$$U^{t^{-1}} BU^{-1} U^{t^{-1}} CU^{-1} = U^{t^{-1}} CU^{-1} U^{t^{-1}} BU^{-1}. \quad (80)$$

Since U is nonsingular

$$BU^{-1} U^{t^{-1}} C = CU^{-1} U^{t^{-1}} B \quad (81)$$

and using (75)

$$BA^{-1}C = CA^{-1}B. \quad (82)$$

Similarly for E', C', and E', B' we get (73).

From the above we can deduce

1) If A, B, C, and E commute we have (73) and (72) are satisfied with T orthogonal (see Appendix D). Hence, for A, B, C, and E commuting is a sufficient condition but not necessary.
2) To get the conditions (20) to the form (72) we invert (20a) and (20b) with matrices L^{-1} and R^{-1} and then use C_o for L^{-1} and use R^{-1} for A.
3) The case of only two matrices is always satisfied as conditions turn into identities as $C = E = 0$. This problem is equivalent to simultaneous reduction to principle axes of two quadratic forms [18]. It is also equivalent to the generalized eigen problem $Bv = \lambda Av$ where A and B are symmetrical real matrices [19]. A similar procedure for the two matrices case is given in [17] using Cholesky factorization and QR decomposition.
4) For the three matrix cases, conditions (73) reduce to one equation only, as $E = 0$. This case is useful for conductor loss only or dielectric loss only. For dielectric loss L^{-1} is taken as matrix A in the above formulation giving (22).
5) The result shows that we can change D_a in (72) to the identity matrix or if required we can equate it to the eigenvalues of A by substituting $T.\ D_a^{-1/2}$ for T where D_a is a diagonal matrix of eigenvalues of the matrix A. The square root will exist since A is strictly positive definite.

Appendix D

For $\{A_i\}$ group of real symmetric matrices then

$$\exists \text{ real orthogonal matrix } T \text{ such that } T^t A_i T \text{ is diagonal}$$
$$\forall\, i \Leftrightarrow A_i A_j = A_j A_i \quad \forall\, i, j.$$

Proof: This tautology is well known in matrix theory and is proven in many references [20], [21].

For implementations, the method in [21] is used which uses induction. The direct approach is to diagonalize each matrix by the *QR Decomposition* using the *Householder* method which is the technique used in most numerical packages for its efficiency, therefore it will be the easiest to apply. However, the *Threshold Jaccobi* method [17] would be more suited since most of the terms in the matrix $T^t A_{n+1} T$ will be zero and it avoids the need for reordering in each step. ■

Appendix E

An example is constructed in this Appendix to produce ZY matrix that is not diagonalizable. Let

$$Z = \begin{bmatrix} 102 + j200 & 1 + j \\ 1 + j & 100 + j202 \end{bmatrix} \quad (83)$$

where Z is complex symmetric and the real and imaginary parts of Z are symmetric real positive definite matrices.

The only eigenvalue of Z is $101 + j201$. The matrix Z is nondiagonalizable with generalized eigenvectors of

$$v_1 = \begin{bmatrix} 1 \\ j \end{bmatrix}$$
$$v_2 = \begin{bmatrix} 1 \\ \dfrac{1+j}{2} \end{bmatrix} \tag{84}$$

such that

$$[v_1 \quad v_2]^{-1} Z [v_1 \quad v_2] = \begin{bmatrix} 101 + j201 & 1 \\ 0 & 101 + j201 \end{bmatrix}. \tag{85}$$

Similarly let

$$Y = \begin{bmatrix} 146 + j175 & -2(1+j) \\ -2(1+j) & 150 + j171 \end{bmatrix} \tag{86}$$

where Y is complex symmetric and the real and imaginary parts of Y are symmetric real positive definite matrices. The only eigenvalue of Y is $148 + j173$. The matrix Y is nondiagonalizable with generalized eigenvectors of

$$w_1 = \begin{bmatrix} 1 \\ j \end{bmatrix},$$

$$\left\{ \begin{array}{cccccc}
\dfrac{1}{\sqrt{n}} & \dfrac{1}{\sqrt{\frac{n}{2}}} & 0 & \cdots & \dfrac{1}{\sqrt{\frac{n}{2}}} & 0 \\
\dfrac{1}{\sqrt{n}} & \dfrac{\cos\left(\frac{2\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left(\frac{2\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\frac{\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} \\
\dfrac{1}{\sqrt{n}} & \dfrac{\cos\left(\frac{4\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left(\frac{4\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\frac{2\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{2\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} \\
\dfrac{1}{\sqrt{n}} & \dfrac{\cos\left(\frac{6\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left(\frac{6\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\frac{3\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{3\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
\dfrac{1}{\sqrt{n}} & \dfrac{\cos\left[\frac{2\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{2\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\pi\frac{(n-1)^2}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{\pi(n-1)^2}{n}\right]}{\sqrt{\frac{n}{2}}}
\end{array} \right\} \tag{96}$$

$$\left\{ \begin{array}{ccccccc}
\dfrac{1}{\sqrt{n}} & \dfrac{1}{\sqrt{n}} & \dfrac{1}{\sqrt{\frac{n}{2}}} & 0 & \cdots & \dfrac{1}{\sqrt{\frac{n}{2}}} & 0 \\
\dfrac{1}{\sqrt{n}} & \dfrac{-1}{\sqrt{n}} & \dfrac{\cos\left(\frac{2\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left(\frac{2\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\frac{\pi(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{\pi(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}} \\
\dfrac{1}{\sqrt{n}} & \dfrac{+1}{\sqrt{n}} & \dfrac{\cos\left(\frac{4\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left(\frac{2\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\frac{2\pi(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{2\pi(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}} \\
\dfrac{1}{\sqrt{n}} & \dfrac{-1}{\sqrt{n}} & \dfrac{\cos\left(\frac{6\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left(\frac{2\pi}{n}\right)}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\frac{3\pi(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{3\pi(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}} \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
\dfrac{1}{\sqrt{n}} & \dfrac{\pm 1}{\sqrt{n}} & \dfrac{\cos\left[\frac{2\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{2\pi(n-1)}{n}\right]}{\sqrt{\frac{n}{2}}} & \cdots & \dfrac{\cos\left[\frac{\pi(n-1)(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}} & \dfrac{\sin\left[\frac{\pi(n-1)(n-2)}{n}\right]}{\sqrt{\frac{n}{2}}}
\end{array} \right\} \tag{97}$$

$$w_2 = \&\begin{bmatrix} 1 \\ \dfrac{-1+j5}{4} \end{bmatrix} \quad (87)$$

such that

$$[w_1 \quad w_2]^{-1} Y [w_1 \quad w_2] = \begin{bmatrix} 148+j173 & 1 \\ 0 & 148+j173 \end{bmatrix}. \quad (88)$$

Hence, the product ZY is

$$ZY = \begin{bmatrix} -20108+j47046 & 175-j283 \\ 175-j283 & -19\,542+j47396 \end{bmatrix} \quad (89)$$

where ZY is also complex symmetric, i.e., in such case the matrices Z and Y commute. Still the only eigenvalue of ZY is $-19\,825+j47221$. The matrix ZY is nondiagonalizable with generalized eigenvectors of

$$x_1 = \begin{bmatrix} 1 \\ j \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ \dfrac{175+j110\,997}{110\,714} \end{bmatrix} \quad (90)$$

such that

$$[x_1 \quad x_2]^{-1} ZY [x_1 \quad x_2] = \begin{bmatrix} -19\,825+j47221 & 1 \\ 0 & -19\,825+j47221 \end{bmatrix}. \quad (91)$$

Appendix F

For evaluation of matrix R, the assumption of $G = 0$ is made and from the formula in [11] for mode i

$$2\alpha^i V^i = RI^i \quad (92)$$

where a_i is the attenuation constant for mode i, V^i and I^i are mode voltages and currents writing (92) in matrix form where α is the diagonal matrix of attenuation constants. M_v and M_i are as in (58)

$$M_v 2\alpha = RM_i$$
$$\Rightarrow R = M_i^{t^{-1}} \alpha M_i^{-1}. \quad (93)$$

Similarly for matrix G

$$\Rightarrow G = M_i^{t^{-1}} \alpha M_v^{-1} \quad (94)$$

where α is the diagonal matrix of attenuation constants.

Appendix G

The procedure in [5] is not repeated. Here only the eigenvalues and the eigenvectors of the general real symmetric Toeplitz matrix with rotational symmetry as in (66) will be given. Note that although such matrix can be diagonalized it will not have n distinct eigenvalues but rather k eigenvalues where k as shown below.

$$\lambda_p = R_0 + 2R_1 \cos\left(\frac{2\pi p}{n}\right) + 2R_2 \cos\left(\frac{4\pi p}{n}\right) + 2R_3 \cos\left(\frac{6\pi p}{n}\right) + \cdots + 2R_k \cos\left(\frac{2k\pi p}{n}\right) \quad \forall\, p = 0 \cdots k \quad (95)$$

where

$$k = \begin{cases} \dfrac{n+2}{2} & n \text{ even} \\ \dfrac{n+1}{2} & n \text{ odd.} \end{cases}$$

The eigenvectors are paired, each two for a single eigenvalue with the exception of the two cases where $p = 0$ and $p = n/2$ for even n, n being the order of the matrix. Hence the orthogonal matrix that congruence diagonalizes M, as in (66), will have a different shape depending whether n is even or odd. For n odd T will be of the form (96), shown on the previous page. The constant vector in first column in the above matrix is due to $p = 0$. For even n at $p = n/2$ the eigenvector corresponding to the sine function will be identically zero and the cosine vector will be a constant vector with alternating sign. For n even T will be of the form (97), shown at the bottom of the previous page.

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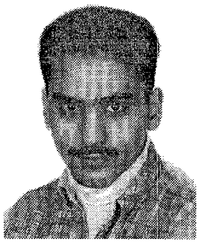

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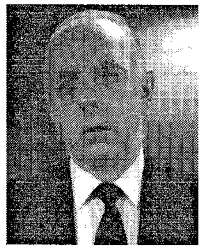

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